

Stats 2 - June 2009

① $H_0: \mu = 768$
 $H_1: \mu \neq 768$ (2 tailed test)

$\bar{x} = 764.8$

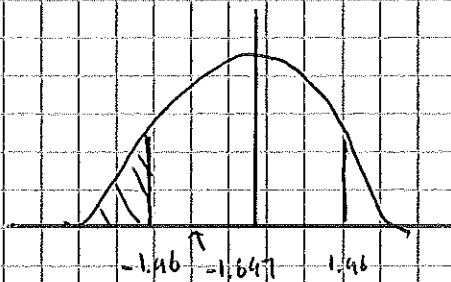
$\sigma = 8$

$n = 18$

we know σ , so can use Z distribution

Test Statistic: $Z = \frac{764.8 - 768}{8/\sqrt{18}} = -1.697\dots$

Critical Value: 5%, 2 tailed: ± 1.96



$-1.697 > -1.96$

\therefore Accept H_0

Not enough evidence at 5% level to support claim that value has changed so not enough evidence to deny Wanne's claim

② a) i) $P(X < 4) = P(X \leq 3) = 0.2650$ (from tables)

ii) $P(X = 4) = \frac{e^{-1.5} \times (1.5)^4}{4!} = 0.0471$

b) i) Let $T =$ Total Letters

$T \sim Po(6.5)$

$P(T > 5) = 1 - P(T \leq 5)$
 $= 1 - 0.3690 = 0.631$

ii) Use Binomial:

7 days ${}^8 C_7 \times (0.631)^7 \times (0.369)^1 = 0.11758$

8 days $(0.631)^8 = 0.02513$

Total Prob = 0.143 (3dp)

c) i) From calc: mean = 8 sd = 4.1096

variance = 16.8

ii) Poisson not a good model as mean and variance are very different.

- ③ H_0 : no association between age and attitude (Independent)
 H_1 : there is an association (non-Independent)

Observed

	Against	Not Against	
16-17	9	2	(1)
18-21	17	10	(2)
22-49	115	96	(20)
50-65	41	34	(15)
Over 65	3	4	(1)
	(85)	(40)	(325)

Expected

	Against	Not Against
16-17	$6\frac{17}{65}$	$4\frac{48}{65}$
18-21	$15\frac{24}{65}$	$11\frac{41}{65}$
22-49	$116\frac{4}{13}$	$88\frac{4}{13}$
50-65	$42\frac{4}{13}$	$32\frac{4}{13}$
Over 65	$3\frac{4}{65}$	$3\frac{1}{65}$

Some Expected values < 5 , so must combine cells:

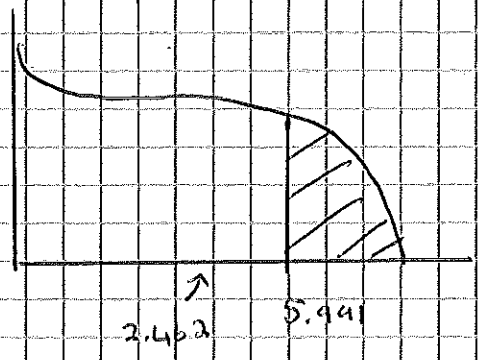
New Expected

			X^2	$\frac{(O-E)^2}{E}$
16-21	21.63	16.37	16-21	0.8925
22-49	116.69	88.31	22-49	0.0245
50+	46.68	35.32	50+	0.1535
				1.1662
				0.0324
				0.2029

Test Statistic (sum of X^2) = 2.462

$v = (3-1) \times (2-1) = 2$

Critical Value (2) 5% = 5.991

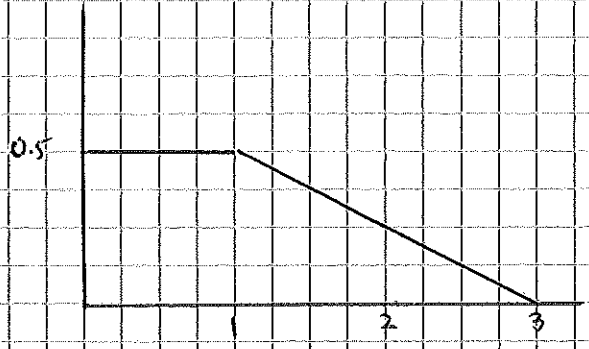


$2.462 < 5.991$

Accept H_0

No evidence at 5% level of an association between age and attitude to school reorganization.

4) a)



b) From graph we can see that $P(X < 1) = 0.5$
 \therefore median = 1

$$c) E(X) = \int_0^3 x f(x) dx$$

$$= \int_0^1 x \cdot \frac{1}{2} dx + \int_1^3 x \cdot \frac{3-x}{4} dx$$

$$= \left[\frac{x^2}{4} \right]_0^1 + \left[\frac{3x^2}{8} - \frac{x^3}{12} \right]_1^3$$

$$= \left[\frac{1}{4} - 0 \right] + \left[\frac{27}{8} - \frac{27}{12} - \frac{3}{8} + \frac{1}{12} \right]$$

$$= \frac{1}{4} + \frac{5}{6} = \frac{13}{12}$$

d) $P(X < 3n - 1) = P(X < 3(\frac{13}{12}) - 1)$
 $= P(X < 2\frac{1}{4})$

From graph, $P(X > 2\frac{1}{4}) = \text{Area of } \Delta$
 $= \frac{1}{2} \times \frac{3}{4} \times \frac{3 - 2\frac{1}{4}}{4} = \frac{9}{128}$
 $\therefore P(X < 2\frac{1}{4}) = 1 - \frac{9}{128} = \frac{119}{128}$

5) a) i) $P(B, B) = 0$, $P(G, G) = \frac{2}{10} \times \frac{1}{9} = \frac{2}{90}$
 $P(Y, Y) = \frac{3}{10} \times \frac{3}{9} = \frac{6}{90}$, $P(R, R) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$
 $\therefore P(2 \text{ same}) = \frac{2}{90} + \frac{6}{90} + \frac{12}{90} = \frac{20}{90} = \frac{2}{9}$

ii) $P(\text{1st blue, 2nd not blue}) = \frac{1}{10} \times \frac{9}{9} = \frac{1}{10}$
 $P(\text{1st not blue, 2nd blue}) = \frac{9}{10} \times \frac{1}{9} = \frac{1}{10}$
 Total Prob = $\frac{1}{10} + \frac{1}{10} = \frac{1}{5}$

b) i)

x	135	145	-45
$P(X=x)$	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{26}{45}$

ii) $E(X) = 135 \times \frac{2}{9} + 145 \times \frac{1}{5} + (-45) \times \frac{26}{45} = 33p$

c) i) $E(Y) = 104 - 3E(X) = 104 - 3 \times 33 = 5p$

Playing 100 times $\rightarrow 100 \times 5p = \pounds 5$ (expected win)

ii) First need $\text{Var}(X)$

$$E(X^2) = 135^2 \times \frac{2}{9} + 145^2 \times \frac{1}{5} + (-145)^2 \times \frac{2}{45}$$

$$= 9425$$

$$\text{Var}(X) = 9425 - 33^2 = 8336$$

$$\text{Var}(Y) = 3^2 \times \text{Var}(X) = 9 \times 8336 = 75,024$$

$$\therefore \text{sd}(Y) = \sqrt{75,024} = 273.9 \text{ p (1dp)}$$

$$= 274 \text{ p (nearest p)}$$

(b) a) i) From calc:

$$\sum x = 348 \quad \bar{x} = 43.5$$

$$\sum x^2 = 15166 \quad s = 2$$

$$n = 8 \quad v = 7$$

t value (1) 95% (2 tailed) = 2.365

$$\text{Confidence Interval} = 43.5 \pm 2.365 \times \frac{2}{\sqrt{8}}$$

$$= 43.5 \pm 1.6723$$

$$= (41.8, 45.2) \quad (1dp)$$

Assumption: weights are normally distributed

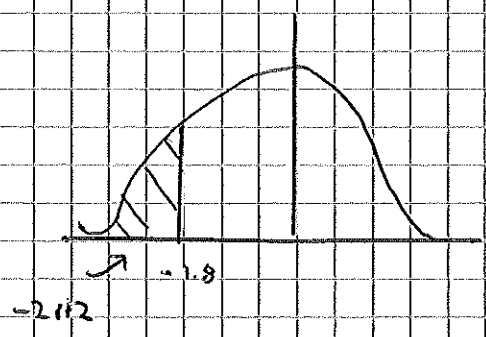
ii) Brian believes mean = 45g. This is just about in the CI, so his belief is justified at 5% level

b) i) $H_0: \mu = 45$

$H_1: \mu < 45$ (1 tailed test)

Test Statistic: $\frac{43.5 - 45}{2/\sqrt{8}} = -2.1213 \dots$

Critical value: $v = 7, 5\%, 1 \text{ tailed} = -1.895$



$$-2.12 < -1.895$$

\therefore Reject H_0

Evidence at 5% level to support Abdul claim that mean content is less than 45g.

ii) Type I Error

Rejected H_0 when H_0 was in fact true.